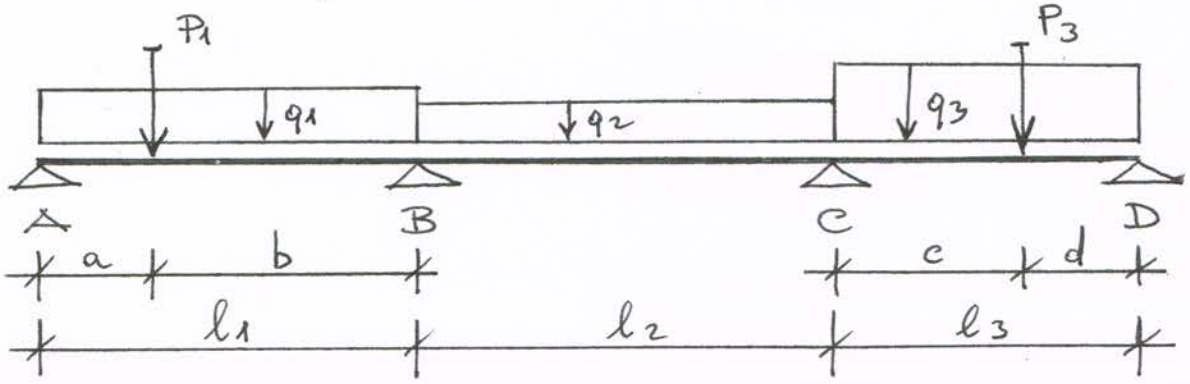
VERIFICA SCRITTA

Risolvere il seguente schema statico a trave continua, determinando le reazioni vincolari e i diagrammi delle sollecitazioni di taglio e momento flettente.



$l_1 = 5{,}00$ mt
$q_1 = 3000$ daN/ml
$P_1 = 1000$ daN $a = 1{,}50$ mt $b = 3{,}50$ mt

$l_2 = 5{,}50$ mt
$q_2 = 2500$ daN/ml
$P_2 =$ /

$l_3 = 4{,}50$ mt
$q_3 = 3200$ daN/ml
$P_3 = 1200$ daN $c = 3{,}00$ mt $d = 1{,}50$ mt

Svolgimento:

$$\widehat{ABC} \rightarrow \begin{cases} M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -6(B_1^* + B_2^*) \\ \widehat{BCD} \rightarrow \quad M_B l_2 + 2M_C(l_2+l_3) + M_D l_3 = -6(C_1^* + C_2^*) \end{cases}$$

Dopo aver scritto le 2 equazioni di CLAPEYRON determiniamo le reazioni fittizie delle travi ausiliarie (MOHR)

$$B_1^* = \frac{q_1 l_1^3}{24} + \frac{P_1 a}{6 l_1}(l_1^2 - a^2) \qquad = 15625 + 1138 \qquad = 16763$$

$$B_2^* = \frac{q_2 l_2^3}{24} \qquad = 17330 \qquad = 17330$$

$$C_1^* = \frac{q_2 l_2^3}{24} \qquad = 17330 \qquad = 17330$$

$$C_2^* = \frac{q_3 l_3^3}{24} + \frac{P_3 d}{6 l_3}(l_3^2 - d^2) \qquad = 12150 + 1200 \qquad = 13350$$

$M_A = 0 \qquad M_D = 0$

$$\begin{cases} 2M_B(5{,}00+5{,}50) + 5{,}50 M_C = -6(16763+17330) \\ 5{,}50 M_B + 2M_C(5{,}50+4{,}50) = -6(17330+13350) \end{cases}$$

$$\begin{cases} 21 M_B + 5{,}50 M_C = -204558 \\ 5{,}50 M_B + 20 M_C = -184080 \end{cases}$$

Risolviamo il sistema con il metodo di sostituzione

$$M_B = \frac{-204558 - 5{,}5\,M_C}{21} = -9740 - 0{,}262\,M_C \qquad (1^a \text{ equazione})$$

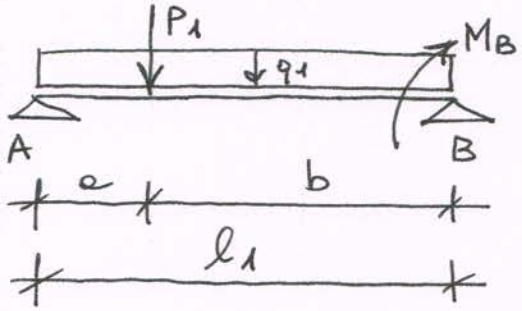
$$5{,}50\left(-9740 - 0{,}262\,M_C\right) + 20\,M_C = -184080 \qquad (2^a \text{ equazione})$$

$$-53570 - 1{,}441\,M_C + 20\,M_C = -184080$$

$$18{,}559\,M_C = -130510 \Longrightarrow \boxed{M_C = -7032}\ \text{daN m}$$

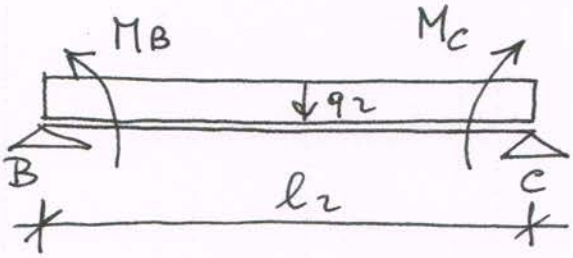
$$M_B = \frac{-204558 + 38676}{21} \Longrightarrow \boxed{M_B = -7899}\ \text{daN m}$$

Determiniamo le reazioni vincolari analizzando le singole campate -



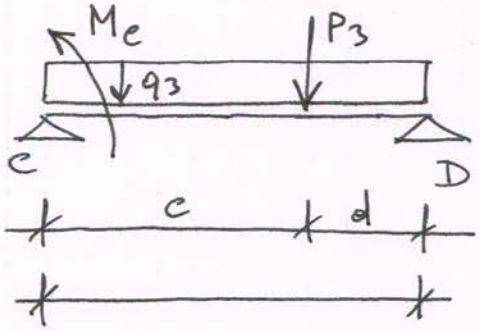
$$V_A = \frac{q_1 l_1}{2} + \frac{P_1 b}{l_1} - \frac{M_B}{l_1} = 6620$$

$$V_B' = \text{"} + \frac{P_1 a}{l_1} + \frac{M_B}{l_1} = 9380$$



$$V_B'' = \frac{q_2 l_2}{2} - \frac{M_C}{l_2} + \frac{M_B}{l_2} = 7032$$

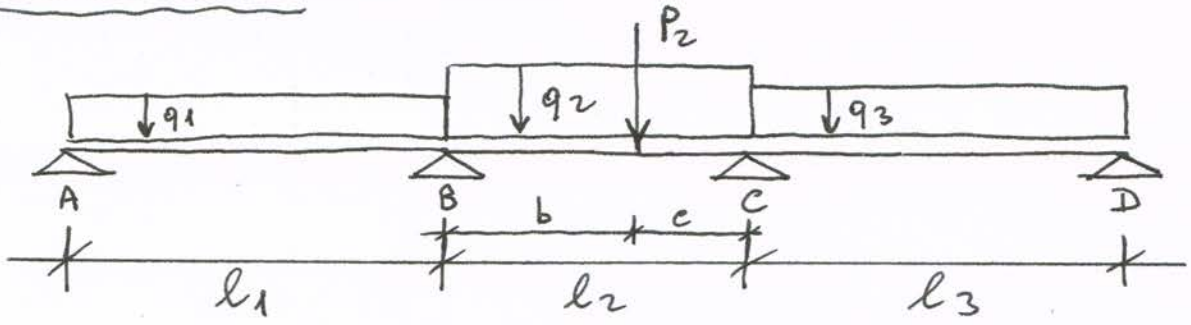
$$V_C' = \frac{q_2 l_2}{2} + \frac{M_C}{l_2} - \frac{M_B}{l_2} = 6717$$



$$V_C'' = \frac{q_3 l_3}{2} + \frac{P_3 d}{l_3} + \frac{M_C}{l_3} = 9163$$

$$V_D = \text{"} + \frac{P_3 c}{l_3} - \frac{M_C}{l_3} = 6437$$

ESERCIZIO



$q_1 = 1000$ daN/ml $\quad l_1 = 6{,}00$ mt

$q_2 = 1800$ " $\quad l_2 = 4{,}00$ mt $\quad P_2 = 500$ daN $\quad b = 2{,}5 \quad e = 1{,}5$

$q_3 = 1500$ " $\quad l_3 = 5{,}00$ mt

$$\widehat{ABC} \rightarrow \begin{cases} M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -6(B_1^* + B_2^*) \\ \widehat{BED} \rightarrow \quad M_B l_2 + 2M_C(l_2+l_3) + M_D l_3 = -6(C_1^* + C_2^*) \end{cases}$$

$$B_1^* = \frac{q_1 l_1^3}{24} = 9000$$

$$B_2^* = \frac{q_2 l_2^3}{24} + \frac{P_2 e}{6 l_2}(l_2^2 - e^2) = 4800 + 430 = 5230$$

$$C_1^* = \frac{q_2 l_2^3}{24} + \frac{P_2 b}{6 l_2}(l_2^2 - b^2) = 4800 + 508 = 5308$$

$$C_2^* = \frac{q_3 l_3^3}{24} = 7812$$

$$\begin{cases} 2M_B(6{,}00+4{,}00) + M_C \times 4 = -6(9000+5230) \\ M_B \times 4 + 2M_C(4{,}00+5{,}00) = -6(5308+7812) \end{cases}$$

$$\begin{cases} 20\,M_B + 4\,M_C = -85380 \\ 4\,M_B + 18\,M_C = -78720 \end{cases}$$

$$M_B = \frac{-85380 - 4M_c}{20} = -4269 - 0{,}2\,M_c$$

$$-4\,(4269 + 0{,}2\,M_c) + 18\,M_c = -78720$$

$$-17076 - 0{,}8\,M_c + 18\,M_c = -78720$$

$$17{,}2\,M_c = -61644$$

$$M_c = -3584 \text{ daNm}$$

$$M_B = -3552 \text{ daNm}$$

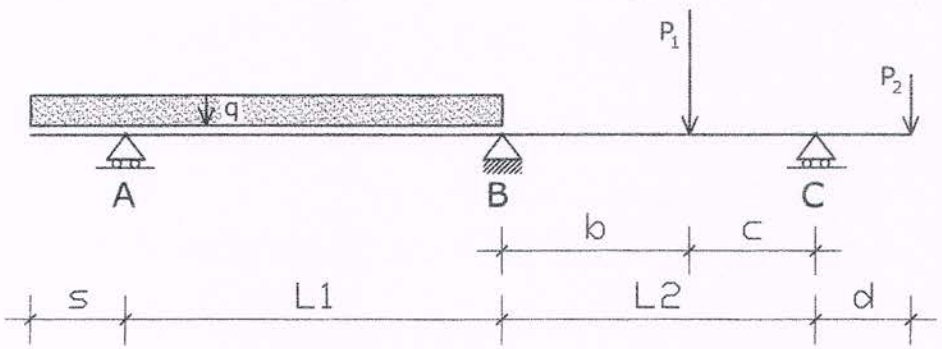
COSTRUZIONI

(Verifica scritta del 26/11/2013)

CLASSE : 4ª Sirio ITG

Allievo : _____ Data : _____

Risolvere il seguente schema statico determinando l'entità delle reazioni vincolari e rappresentando i diagrammi delle caratteristiche di sollecitazione (**T – M**) .



$L_1 = 6{,}00$ m $\quad L_2 = 5{,}00$ m

b = 3,00 m $\quad$ c = 2,00 m

s = 1,50 m $\quad$ d = 1,50 m

q = 3500 daN/m

$P_1 = 6000$ daN

$P_2 = 2000$ daN

SVOLGIMENTO :

Trattasi di una trave continua a 2 campate con sbalzi alle estremità

Il sistema è 1 volta iperstatico

INCOGNITA IPERSTATICA = M_B

Per determinare il momento M_B applicheremo e risolveremo l'equazione dei 3 momenti (CLAPEYRON).

Determiniamo gli effetti degli sbalzi sulle campate adiacenti :

Sbalzo a sinistra $Q = qs = 3500 \times 1,50 = 5250$ daN

$$M_A = -\frac{qs^2}{2} = -\frac{3500 \times 1,50^2}{2} = -3940 \text{ daNm}$$

Sbalzo a destra $P_2 = 2000$ daN

$$M_C = -P_2 d = -2000 \times 1,50 = -3000 \text{ daNm}$$

Esplicitiamo l'equazione dei 3 momenti relativamente alle coppie di campate A-B e B-C

$$M_A l_1 + 2M_B (l_1 + l_2) + M_C l_2 = -6 \left(R_1^* + R_2^*\right)$$

$$R_1^* = \frac{q l_1^3}{24} = \frac{3500 \times 6^3}{24} = 31500$$

$$R_2^* = \frac{P_1 c}{6 l_2}\left(l_2^2 - c^2\right) = \frac{6000 \times 2}{6 \times 5}\left(5^2 - 2^2\right) = 8400$$

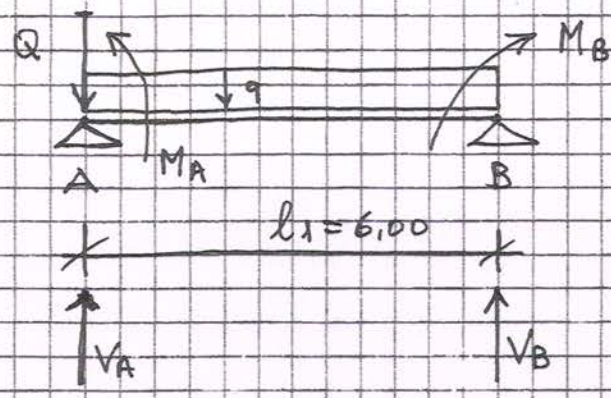
$$-3940 \times 6 + 2M_B (6+5) - 3000 \times 5 = -6 (31500 + 8400)$$

$$22 M_B = 23640 + 15000 - 239400$$

$$M_B = -\frac{200760}{22} = -9125 \text{ daNm}$$

Avendo determinato l'incognita iperstatica M_B è possibile risolvere le singole campate determinando le reazioni vincolari e i diagrammi delle caratteristiche di sollecitazione.

CAMPATA A – B



$M_A = 3940$ daNm

$M_B = 9125$ "

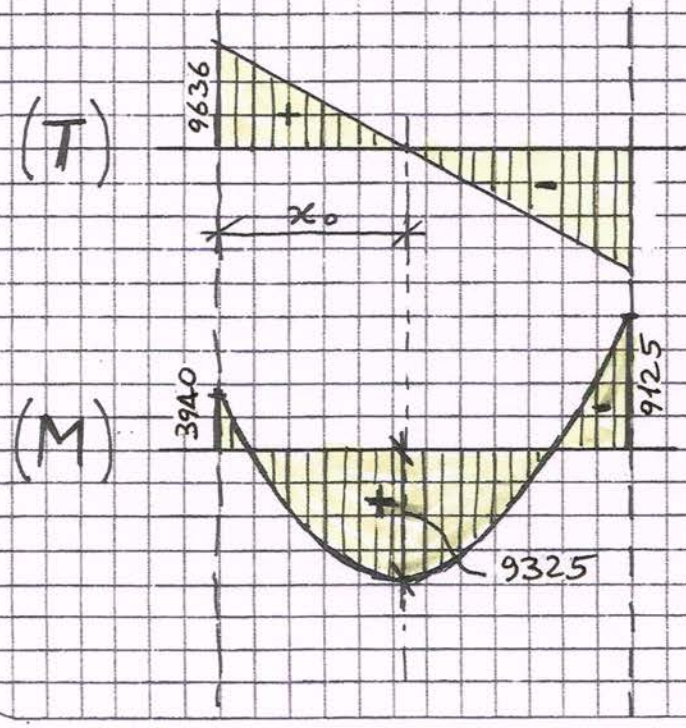
$q = 3500$ daN/m

$Q = 5250$ daN

$$V_A = Q + \frac{M_A}{l_1} + \frac{ql_1}{2} - \frac{M_B}{l_1} = 5250 + 657 + 10500 - 1521 = \underline{14886}\ \text{daN}$$

$$V_B = -\frac{M_A}{l_1} + \frac{ql}{2} + \frac{M_B}{l_1} = -657 + 10500 + 1521 = \underline{11364}\ \text{N}$$

Il diagramma del taglio sarà lineare mentre quello del momento avrà andamento parabolico:



$T_A = V_A - Q = 9636$ daN

$T_B = -V_B = -11364$ daN

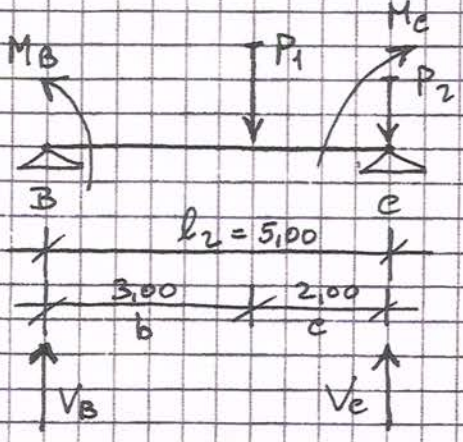
$$x_0 = \frac{V_A - Q}{q} = 2{,}75\ \text{m}$$

$$M(x_0) = (V_A - Q)x_0 - M_A - \frac{q x_0^2}{2} =$$

$$= 26499 - 3940 - 13234 =$$

$$= 9325\ \text{daNm}$$

CAMPATA B-C



$M_B = 9125$ daNm

$M_C = 3000$ "

$P_1 = 6000$ daN

$P_2 = 2000$ daN

$$V_B = \frac{M_B}{l_2} + P_1\frac{c}{l_2} - \frac{M_C}{l_2} = 1825 + 2400 - 600 = 3625 \text{ daN}$$

$$V_C = -\frac{M_B}{l_2} + P_1\frac{b}{l_2} + \frac{M_C}{l_2} + P_2 = -1825 + 3600 + 600 + 2000 = 4375 \text{ daN}$$

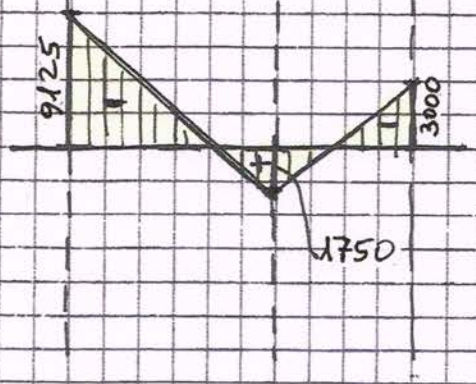
Il diagramma del taglio sarà lineare a tratti costanti mentre quello del momento avrà andamento lineare secondo una spezzata

$T_B = +V_B = +3625$ daN

$T_C = -V_C + P_2 = -2375$ daN

(T)

(M)



$M_{max} = V_B \times b - M_B =$

$= 10875 - 9125 =$

$= 1750$ daNm

COSTRUZIONI

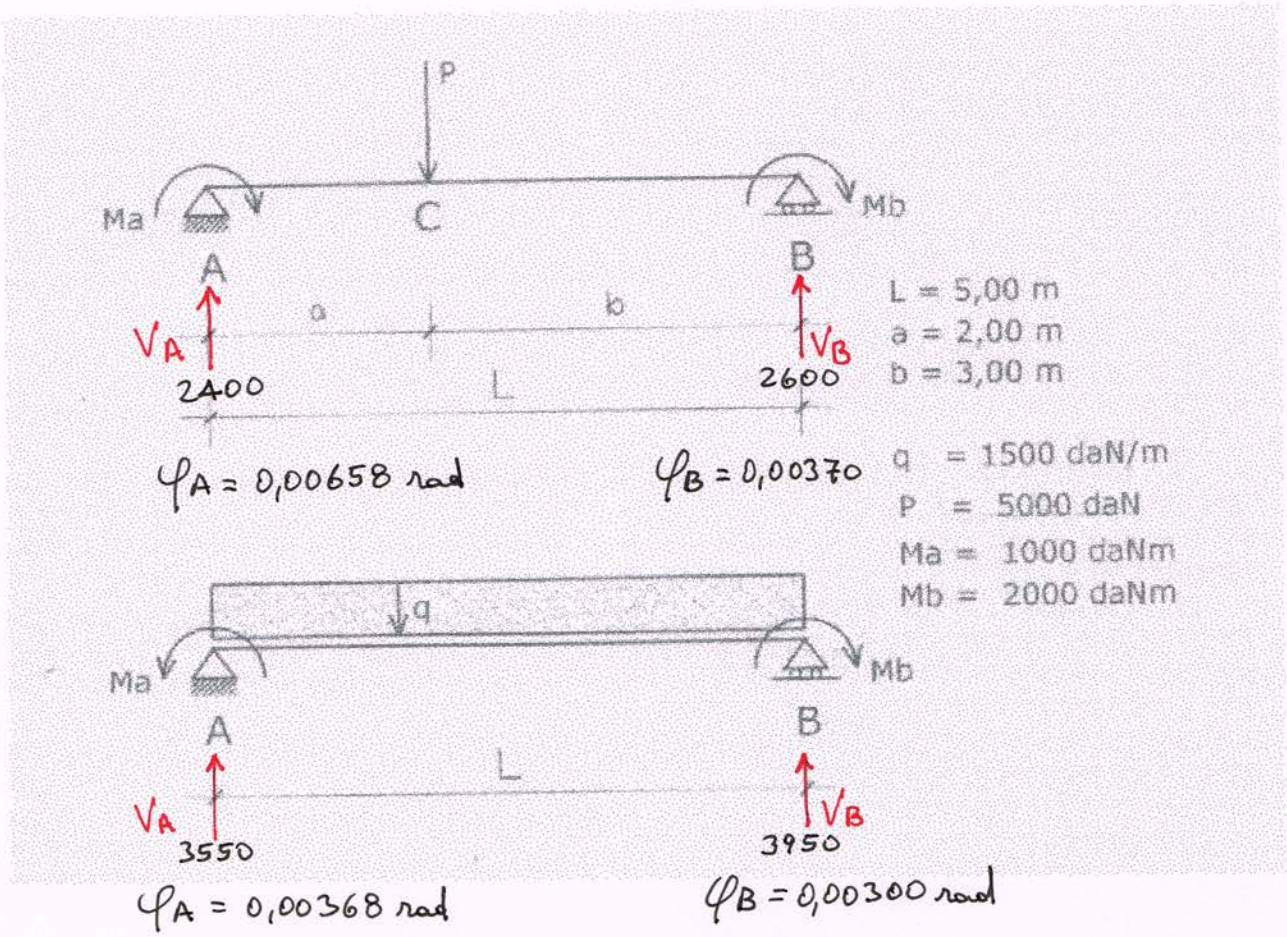
(Verifica scritta del 23/10/2012)

CLASSE 4ª Sirio ITG

Allievo : _____ Data : _____

Esercizio 1 :

Determinare le reazioni vincolari e i diagrammi delle sollecitazioni di Taglio e Momento flettente dei seguenti schemi statici

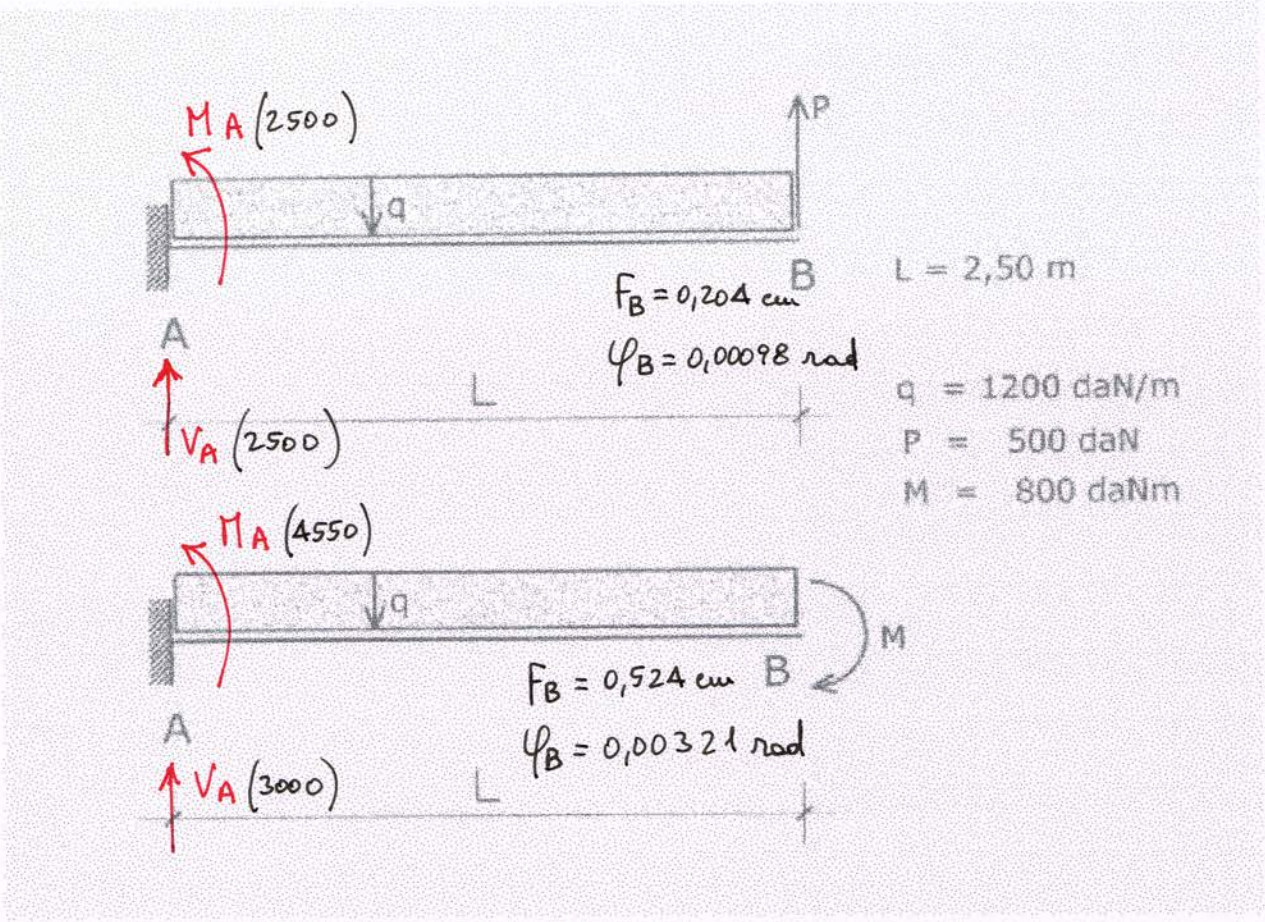


Ipotizzando di realizzare la trave con un profilo IPE 270 (J=5790 cm^4) in acciaio determinare le rotazioni in A e in B

ACCIAIO **E** = 2100000 daN/cm^2

Esercizio 2 :

Determinare le reazioni vincolari e i diagrammi delle sollecitazioni di Taglio e Momento flettente dei seguenti schemi statici



Ipotizzando di realizzare la trave in legno con una sezione 30x45 determinare l'abbassamento e la rotazione in B

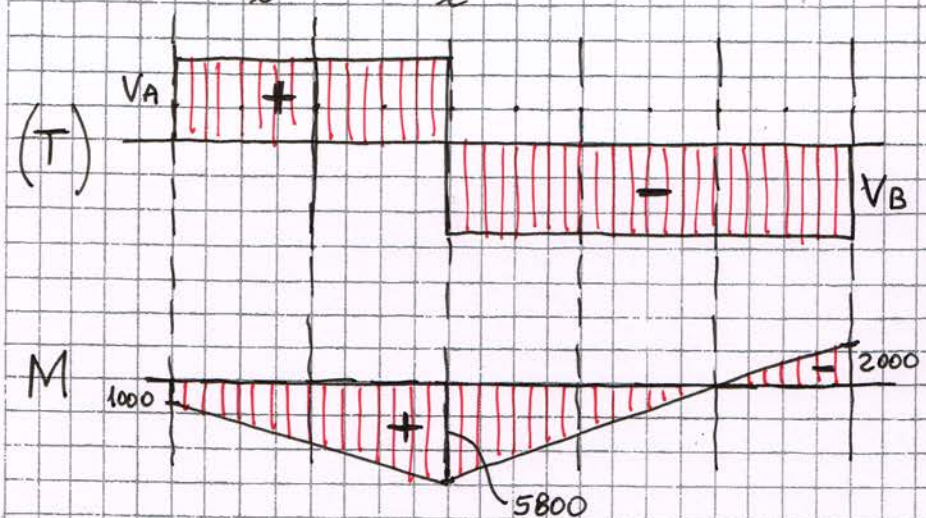
LEGNO $\mathbf{E} = 70000\ \text{daN/cm}^2$

SVOLGIMENTO

Esercizio 1:

$$V_A = \frac{Pb}{\ell} - \frac{M_a + M_B}{\ell} = 3000 - 600 = 2400 \text{ daN}$$

$$V_B = \frac{Pa}{\ell} + \frac{M_a + M_B}{\ell} = 2000 + 600 = 2600 \text{ daN}$$



$T(A) = V_A = 2400$

$T(B) = -V_B = -2600$

$M_A = +M_a = 1000$

$M_B = -M_b = -2000$

$M_C = V_A \cdot a + M_a = 5800$

$$\varphi_A = \frac{M_a \ell}{3EJ} + \frac{Pb}{6\ell EJ}\left(\ell^2 - b^2\right) - \frac{M_b \ell}{6EJ} =$$

$$= \frac{1\cdot 10^5 \times 500}{3EJ} + \frac{5000 \times 300}{6 \times 500\, EJ}\left(500^2 - 300^2\right) - \frac{2\times 10^5 \times 500}{6EJ} =$$

$$= 0{,}00137 + 0{,}00658 - 0{,}00137 = \underline{0{,}00658 \text{ rad}}$$

$$\varphi_B = \frac{M_a \ell}{6EJ} + \frac{Pa}{6\ell EJ}\left(\ell^2 - a^2\right) - \frac{M_b \ell}{3EJ} =$$

$$= \frac{1\times 10^5 \times 500}{6EJ} + \frac{5000 \times 200}{6 \times 500\, EJ}\left(500^2 - 200^2\right) - \frac{2\times 10^5 \times 500}{3EJ} =$$

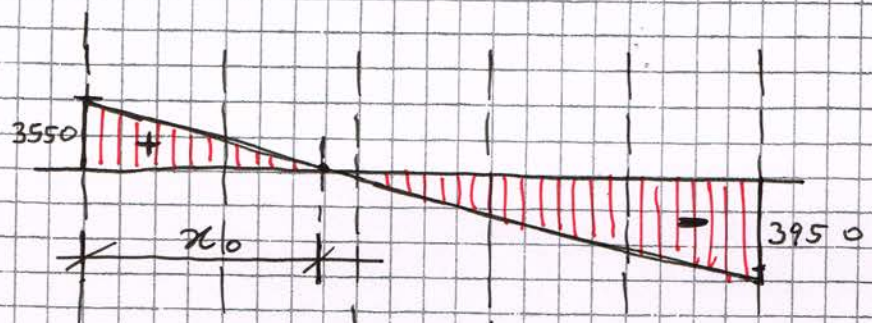
$$= 0{,}00068 + 0{,}00576 - 0{,}00274 = \underline{0{,}00370 \text{ rad}}$$

Esercizio 1 :

$$V_A = \frac{ql}{2} + \frac{M_A}{l} - \frac{M_b}{l} = 3750 + 200 - 400 = 3550 \text{ daN}$$

$$V_B = \frac{ql}{2} - \frac{M_A}{l} + \frac{M_b}{l} = 3750 - 200 + 400 = 3950 \text{ daN}$$

(T)

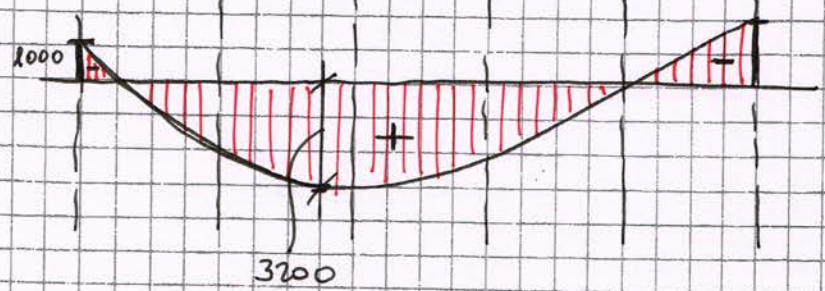


$T_A = V_A = 3550$

$T_B = -V_B = -3950$

$x_0 = \frac{V_A}{q} = 2{,}37$

(M)



$M_A = -M_a = -1000$

$M_B = -M_b = -2000$

$M(x_0) = V_A \cdot x_0 - M_a - \frac{q x_0^2}{2} =$

$= 3200 \text{ daNm}$

$$\varphi_A = -\frac{M_a l}{3EJ} - \frac{M_b l}{6EJ} + \frac{q l^3}{24EJ} =$$

$$= -\frac{1\times10^5 \times 500}{3EJ} - \frac{2\times10^5 \times 500}{6EJ} + \frac{15 \times 500^3}{24EJ} =$$

$$= -0{,}00137 - 0{,}00137 + 0{,}00642 = \underline{0{,}00368 \text{ rad}}$$

$$\varphi_B = -\frac{M_a l}{6EJ} - \frac{M_b l}{3EJ} + \frac{q l^3}{24EJ} =$$

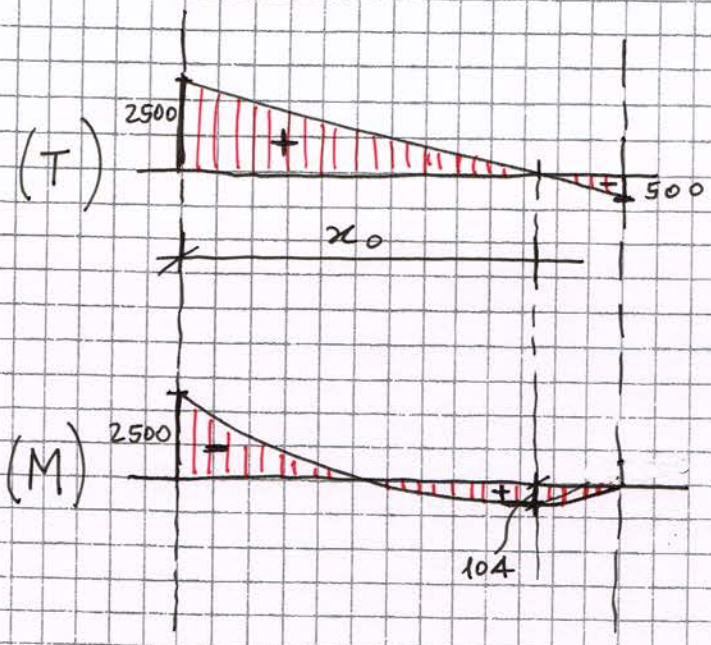
$$= -0{,}00068 - 0{,}00274 + 0{,}00642 = \underline{0{,}00300 \text{ rad}}$$

Esercizio 2

$\left(J = \frac{1}{12} 30 \times 45^3 = 227812 \text{ cm}^4 \right)$

$V_A = ql - P = 1200 \times 2{,}50 - 500 = 2500 \text{ daN}$

$M_A = \frac{ql^2}{2} - Pl = 1200 \times \frac{250^2}{2} - 500 \times 2{,}50 = 2500 \text{ daNm}$



$T_A = V_A = 2500$

$T_B = -P = -500$

$x_0 = \frac{V_A}{q} = 2{,}08$

$M_A = -M_A = -2500$

$M_B = 0$

$M(x_0) = V_A \cdot x_0 - M_A - \frac{q x_0^2}{2} =$

$= 104 \text{ daNm}$

$$f_B = \frac{ql^4}{8EJ} - \frac{Pl^3}{3EJ} = \frac{12 \times 250^4}{8EJ} - \frac{500 \times 250^3}{3EJ} =$$

$$= 0{,}367 - 0{,}163 = \underline{0{,}204 \text{ cm}}$$

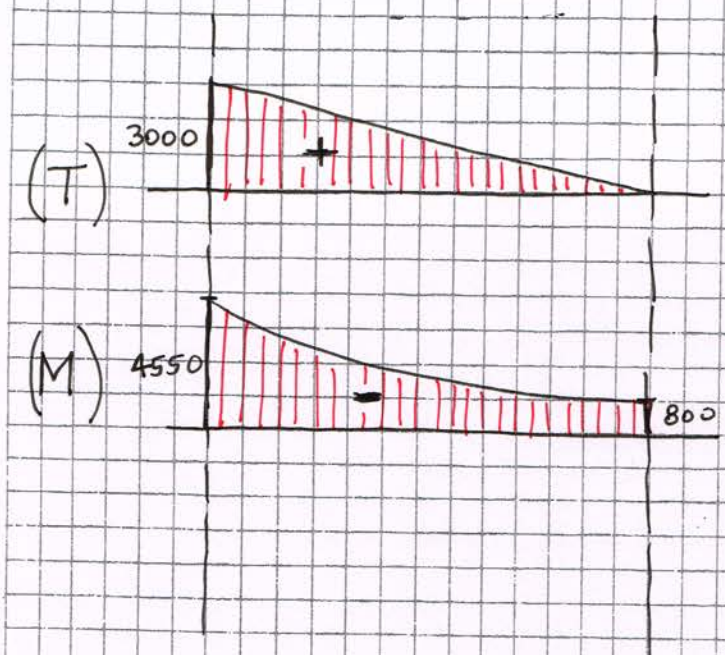
$$\varphi_B = \frac{ql^3}{6EJ} - \frac{Pl^2}{2EJ} = \frac{12 \times 250^3}{6EJ} - \frac{500 \times 250^2}{2EJ} =$$

$$= 0{,}00196 - 0{,}00098 = \underline{0{,}00098 \text{ rad}}$$

Esercizio 2 $\left(J = 227812\ cm^4\right)$

$V_A = ql = 3000\ daN$

$M_A = \frac{ql^2}{2} + M = 3750 + 800 = 4550\ daNm$



$T(A) = V_A = +3000$

$T(B) = 0$

$M_A = -M_A = -4550\ daNm$

$M_B = -M = -800$ "

$$f_B = \frac{ql^4}{8EJ} + \frac{Ml^2}{2EJ} = \frac{12 \times 250^4}{8EJ} + \frac{80000 \times 250^2}{2EJ} =$$

$$= 0{,}367 + 0{,}157 = \underline{0{,}524\ cm}$$

$$\varphi_B = \frac{ql^3}{6EJ} + \frac{Ml}{EJ} = \frac{12 \times 250^3}{6EJ} + \frac{80000 \times 250}{EJ} =$$

$$= 0{,}00196 + 0{,}00125 = \underline{0{,}00321\ rad}$$